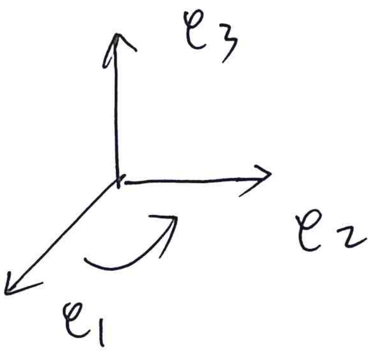
tutorial notes. 1.

1. Standard orientation

$\vec{e}_i = (0, \cdots, 0, 1, 0, \cdots, 0)$
$\uparrow$ ith coordinate

$\Rightarrow \{\vec{e}_1, \vec{e}_2, \cdots, \vec{e}_n\}$ forms a standard ordered basis

(corresponding to the standard orientation)
$\downarrow$
right hand rule.



(1) $\{\vec{e}_1, \vec{e}_3, \vec{e}_2\} = -\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$.
NS (not standard)

(2) $\{\vec{e}_2, \vec{e}_3, \vec{e}_1\} = \{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$.
S

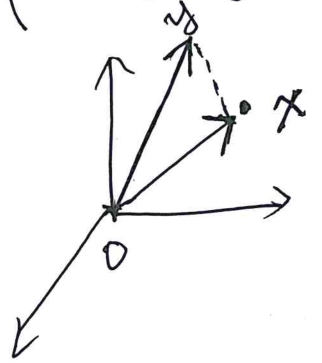
(3) $\{\vec{e}_3, \vec{e}_1, \vec{e}_2\} = \{\vec{e}_1, \vec{e}_3, \vec{e}_3\}$.

reason: change $\vec{e}_3, \vec{e}_1$ position $\rightarrow -\{\vec{e}_1, \vec{e}_3, \vec{e}_2\}$

change $\vec{e}_3, \vec{e}_2$ position $\longrightarrow -(-\{\vec{e}_1, \vec{e}_2, \vec{e}_3\})$.

$= \{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ by basic property of determinant.

2. the n-dimension vector forms a well corresponding relation with the point in n-dimension space. we can treat the point x in $\mathbb{R}^n$ with the vector from O to x as the same guy.



so for two points in $\mathbb{R}^n$, x and y. we know the distance of them is $\| x - y \|$

when we set $x = (x_1, x_2, \cdots, x_n)$

$y = (y_1, y_2, \cdots, y_n)$

we ~~see~~ review the distance formula.

$$d(x,y) = \| x - y \| = \sqrt{(x_1-y_1)^2 + \cdots + (x_n - y_n)^2}$$

that is a natural generalization of the

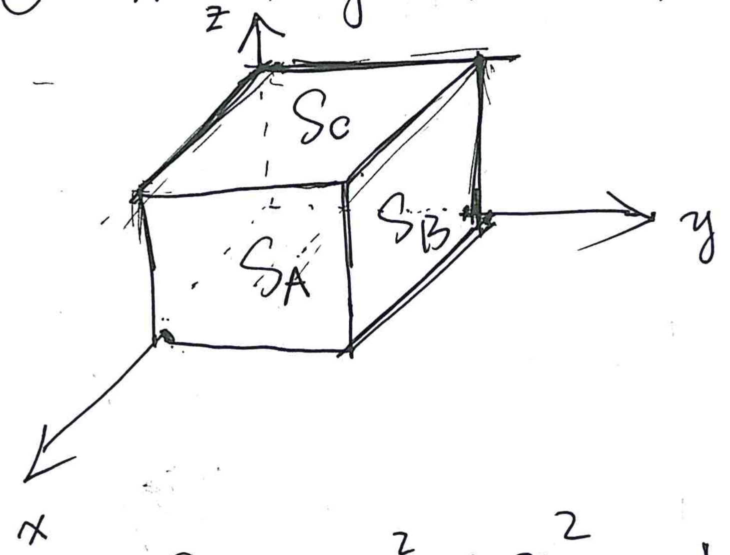
2-dim distance formula to n-dim.

the tool you have known is the so-called

Inner product !

3. Some exercise about drawing graph.

① $x=1$, $y=1$, $z=1$ respectively (first quadrant



$x=1$, S_A

y ~~z~~ $=1$, S_B

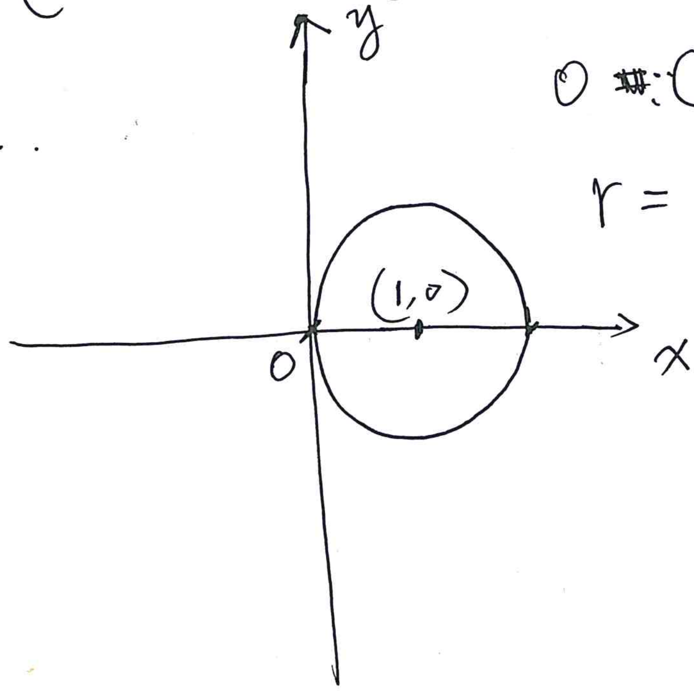
$z=1$, S_C.

② $(x-1)^2+y^2=1$

$n=2$.

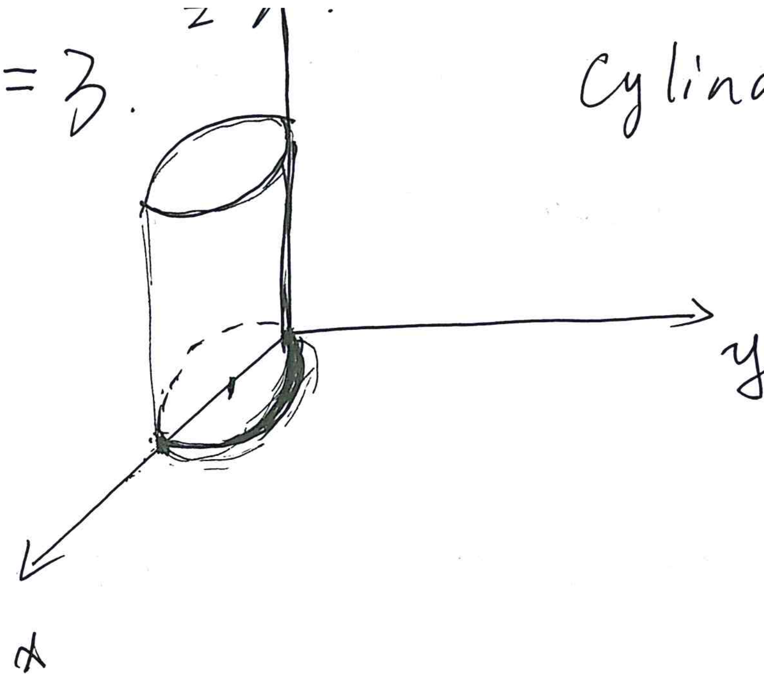
O ~~#~~: $(1,0)$

$r=1$.



$n = 3.$

Cylinder surface.



③ find the centre and radii of surface. $2x^2+2y^2+2z^2+x+y+z=9$.

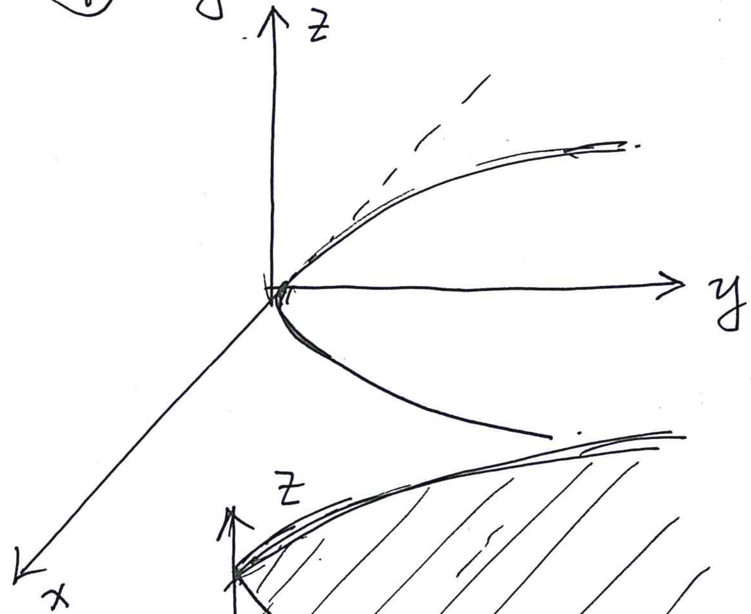
$$\Rightarrow 2\left(x^2+\frac{1}{2}x\right)+2\left(y^2+\frac{1}{2}y\right)+2\left(z^2+\frac{1}{2}z\right)=9$$

$$\Rightarrow 2\left(x+\frac{1}{4}\right)^2+2\left(\cancel{x}\,y+\frac{1}{4}\right)^2+2\left(z+\frac{1}{4}\right)^2=69/8$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2+\left(y+\frac{1}{4}\right)^2+\left(z+\frac{1}{4}\right)^2=\left(\frac{\sqrt{69}}{4}\right)^2$$

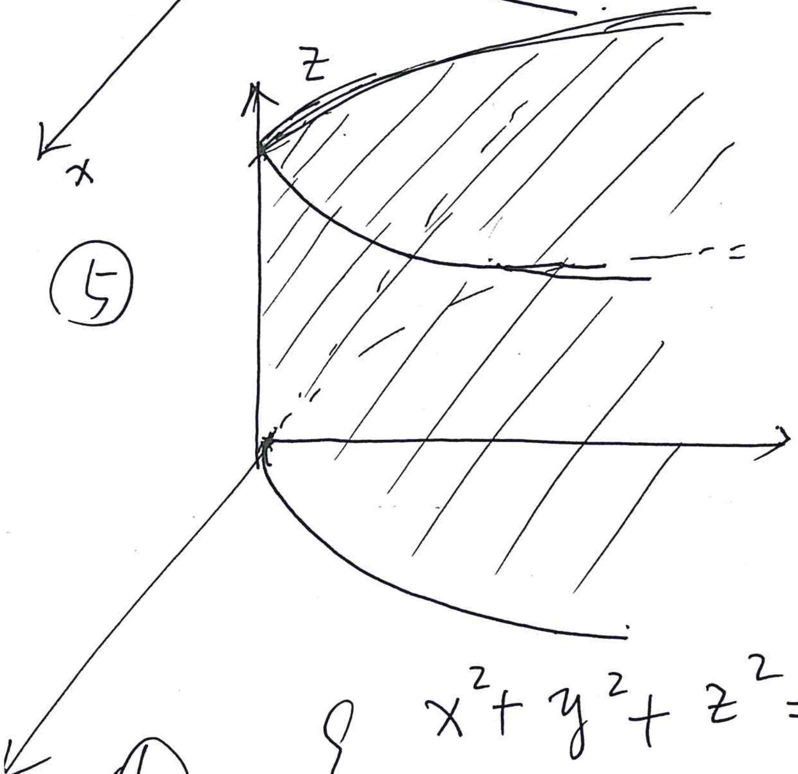
$$O:\left(-\frac{1}{4},-\frac{1}{4},-\frac{1}{4}\right),\quad r=\left(\frac{\sqrt{69}}{4}\right)^2$$

④ $y = x^2,\ z = 0.$

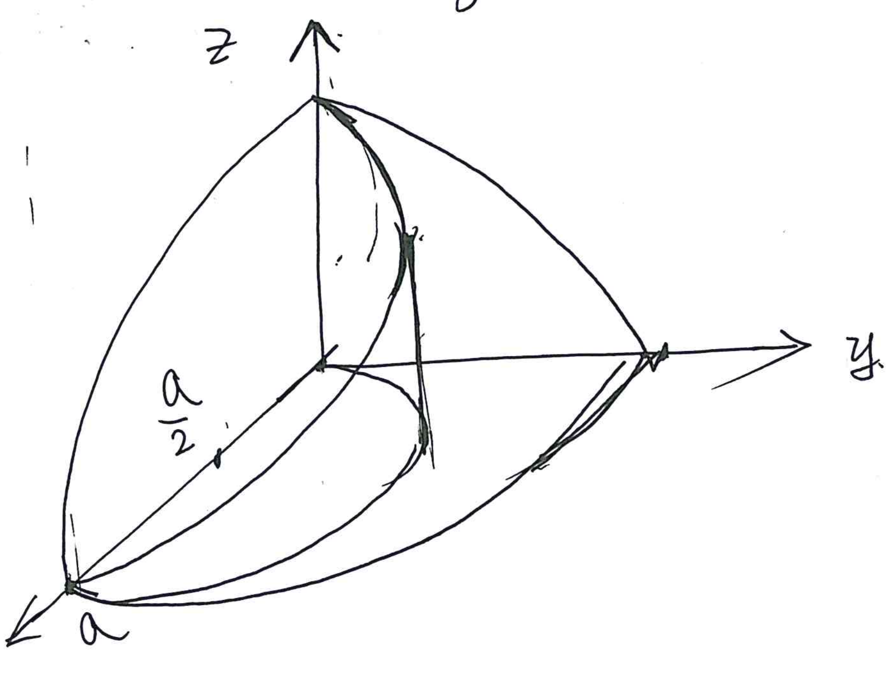


$y \geq x^2,\ z \geq 0.$

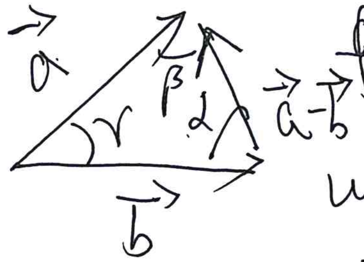
⑤



⑥ $\begin{cases} x^2+y^2+z^2=a^2 \\ x^2+y^2=ax \end{cases}$ (first quadrant)



Complement for inner product and norm.



for a triangle. we have the <u>sin law</u> as follow:

$$\frac{\|\vec{a}\|}{\sin\alpha} = \frac{\|\vec{b}\|}{\sin\beta} = \frac{\|\vec{a}-\vec{b}\|}{\sin\gamma}.$$

$(\alpha+\beta+\gamma=\pi)$

$\|x\| = \sqrt{\langle x,x\rangle}$.

norm: three important property (or define method).

for Euclid norm especially: $x \in \mathbb{R}^n$

1. $\|x\| = 0 \iff x = 0$ $\qquad x = \sqrt{x_1^2+x_2^2+\cdots x_n^2}$

2. $\|\lambda x\| = |\lambda|\cdot\|x\|$

3. $\|x+y\| \leq \|x\| + \|y\|$

~~Cauchy-Schwarz~~

a generalization of Cauchy-Schwarz inequality.

<u>Holder inequality</u>: $x = (x_1, x_2, \cdots, x_n) \in \mathbb{R}^n$

$y = (y_1, y_2, \cdots, y_n) \in \mathbb{R}^n$

$$\underline{|\langle x, y\rangle| \leq \|x\|_p \cdot \|y\|_q .}$$

in which. $\|x\|_p = (|x_1|^p + |x_2|^p + \cdots + |x_n|^p)^{\frac{1}{p}}$

L^p-norm for $\mathbb{R}^n$

$\|y\|_q = (|y_1|^q + |y_2|^q + \cdots + |y_n|^q)^{\frac{1}{q}}$

~~and~~ . $\langle x, y\rangle = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n$

$\frac{1}{p} + \frac{1}{q} = 1$, (p, q) called the Holder index

$(2,2)$ corresponding Cauchy-Schwarz inequality.

$(p \geq 1, q > 1)$